## 8.8 – Operations with Rational Expressions

*Objectives:*

1. Learn to add, subtract, multiply and divide rational expressions.
2. Review factoring.
3. Simplify rational expressions in order to find x-intercepts, vertical asymptotes, and holes.

**Example 1:** Add the expressions below and write as a single sum:

$$\frac{x-1}{(x-2)(x+1)}+\frac{x}{(x+1)(x-1)}$$

$\frac{(x-1)}{(x-1)}$ $\frac{(x-2)}{(x-2)}$

$$\frac{x^2-2x+1+x^2-2x}{(x-1)(x-2)(x+1)}$$

$$\boxed{\frac{2x^2-4x+1}{(x-1)(x-2)(x+1)}}$$

**Example 2:** Add the expressions below and write as a single sum:

$$\frac{4}{x^2-2x-3}+\frac{x-2}{x^2+2x+1}$$

$\frac{(x+1)}{(x+1)}$ $\frac{(x-3)}{(x-3)}$

$$\frac{4}{(x-3)(x+1)}+\frac{x-2}{(x+1)(x+1)}$$

$$\frac{4x+4}{(x+1)(x-3)(x+1)}+\frac{x^2-5x+6}{(x+1)(x+1)(x+3)}$$

$$\frac{x^2-x+10}{(x+1)(x-3)(x+1)}$$

**Example 3:** Write the differences as a single rational expression.

$$\frac{x+2}{(x-3)(x+4)} - \frac{5}{x+1}$$

$\frac{(x+1)}{(x+1)}$ $\qquad$ $\frac{(x-3)(x+4)}{(x-3)(x+4)}$

$$\frac{x^2+3x+2}{(x+1)(x-3)(x+4)} + \frac{-5(x-3)(x+4)}{(x+1)(x-3)(x+4)}$$

$$\frac{x^2+3x+2}{(x+1)(x-3)(x+4)} + \frac{(-5x+15)(x+4)}{(x+1)(x-3)(x+4)}$$

$$\frac{x^2+3x+2-5x^2-20x+15x+60}{(x+1)(x-3)(x+4)}$$

$$\frac{-4x^2-2x+62}{(x+1)(x-3)(x+4)}$$

**Example 4:** Write the differences as a single rational expression:

$$\frac{4}{x^2-49} - \frac{x}{(x+7)(x-1)}$$

$\frac{(x-1)}{(x-1)} \cdot \frac{4}{(x+7)(x-7)} - \frac{x}{(x+7)(x-1)} \cdot \frac{(x-7)}{(x-7)}$

$$\frac{4x-4}{(x-1)(x+7)(x-7)} + \frac{-x^2+7x}{(x+7)(x-1)(x-7)}$$

$$\frac{-x^2+11x-4}{(x-1)(x+7)(x-7)}$$

**Example 5:** Multiply: $\dfrac{(x+2)}{(x-3)} \cdot \dfrac{(x+1)(x-3)}{(x-1)(x^2-4)}$

$\dfrac{(x+2)}{(x-3)} \cdot \dfrac{(x+1)(x-3)}{(x-1)(x+2)(x-2)}$

$\boxed{\dfrac{x+1}{(x-1)(x-2)}}$

**Example 6:** Multiply: $\dfrac{x^2+2x-15}{2x^2+9x-5} \cdot \dfrac{4x^2-1}{2x^2-5x-3}$

$\dfrac{(x+5)(x-3)}{(2x-1)(x+5)} \cdot \dfrac{(2x+1)(2x-1)}{(2x+1)(x-3)}$

$\boxed{1}$

**Example 7:** Divide: $\dfrac{x^2-1}{\dfrac{x^2+5x+6}{x^2-3x+2}}$

$\dfrac{x+3}{(x+1)(x-1)} \div \dfrac{(x-2)(x-1)}{(x+3)}$

$\dfrac{(x+1)(x-1)}{(x+3)(x+2)} \cdot \dfrac{(x+3)}{(x-2)(x-1)}$

$\boxed{\dfrac{x+1}{x^2-4}}$

**Example 8:** $\dfrac{9x^2+6x}{2x-1} \div \dfrac{6x^2+x-2}{4x^2-4x+1}$

$\dfrac{3x(3x+2)}{2x-1} \div \dfrac{(3x+2)(2x-1)}{(2x-1)(2x-1)}$

$\dfrac{3x(3x+2)}{2x-1} \cdot \dfrac{(2x-1)(2x-1)}{(2x-1)(3x+2)}$

$\boxed{3x}$